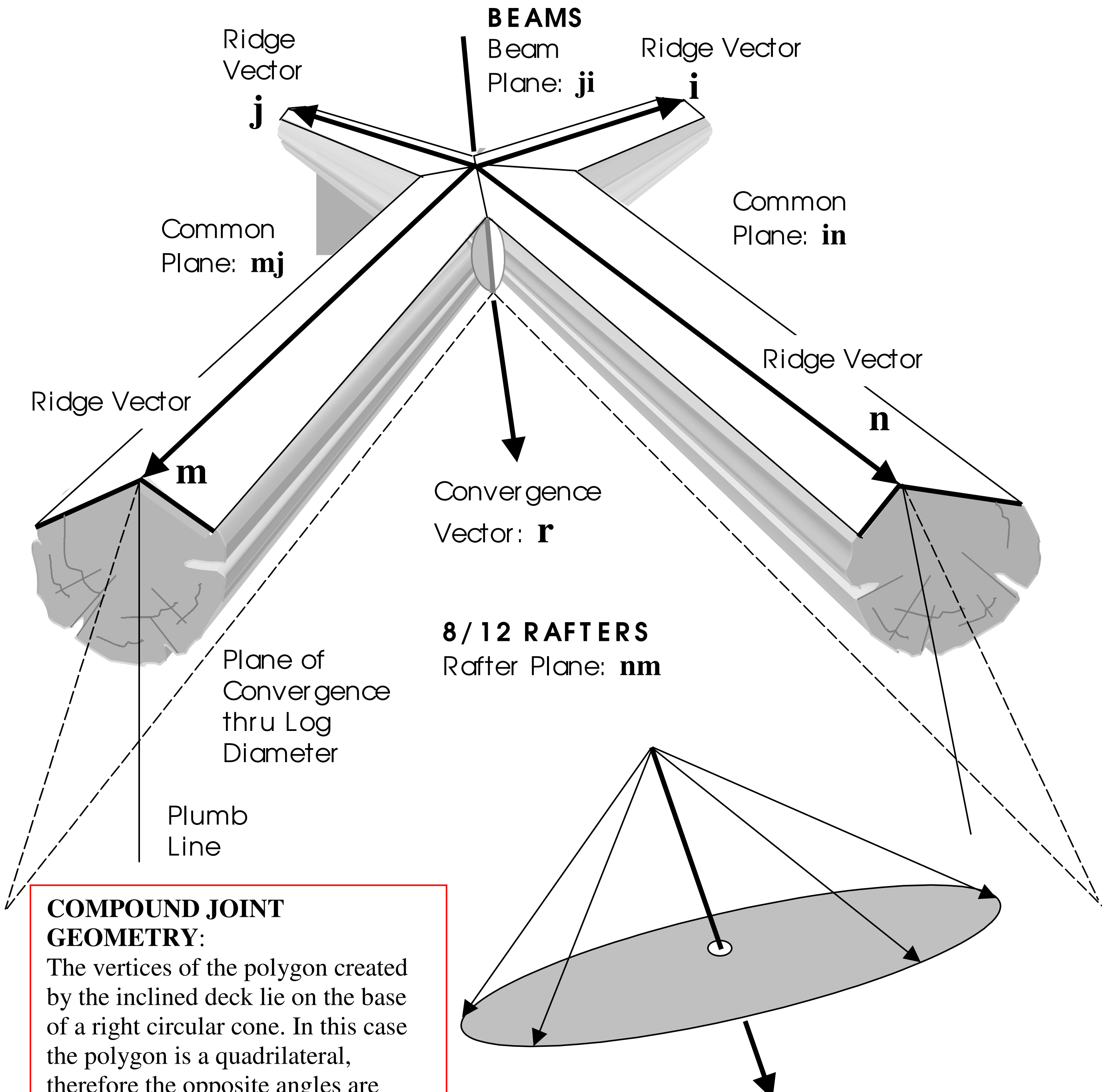
<u>COMPOUND JOINT:</u>

Alternate Vector Analysis Solution

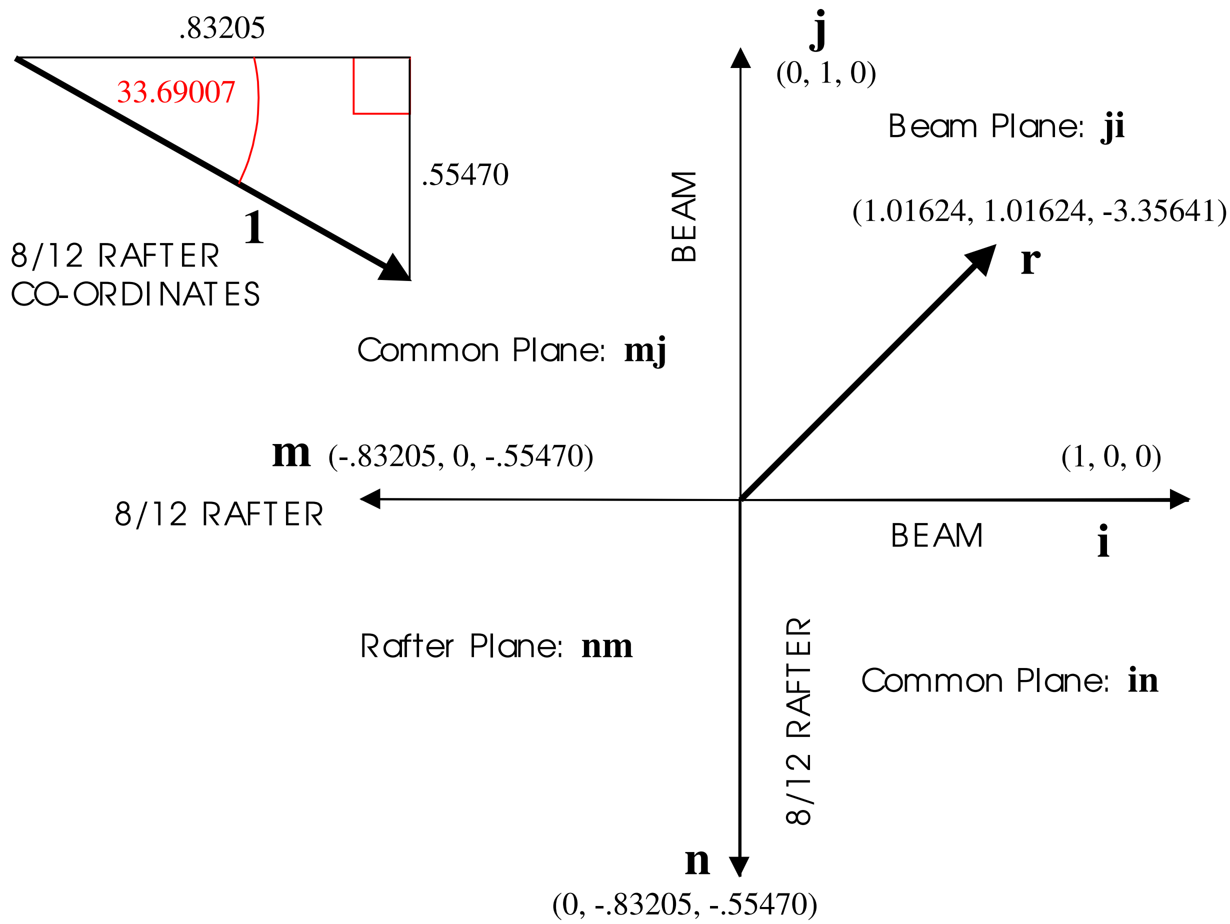


COMPOUND JOINT GEOMETRY:
The vertices of the polygon created by the inclined deck lie on the base of a right circular cone. In this case the polygon is a quadrilateral, therefore the opposite angles are supplementary.
<u>Corollary</u>: Since any triangle may be circumscribed, three members always converge.

Convergence vector, **r**, lies on the altitude of the cone, and meets the Ridge vectors at the vertex. The Ridge vectors are of equal magnitude.

VECTOR DIAGRAM:

Co-ordinates of Points for Angle Calculations



(**a**, **b**) means the angle between vectors **a** and **b**, solved using the equation:

$(\mathbf{a}, \mathbf{b}) = \arccos \pm \left[(\mathbf{a} \cdot \mathbf{b}) / |\mathbf{a}|\,|\mathbf{b}|\right]$

$\mathbf{r} = \mathbf{a} \times \mathbf{b}$ may be evaluated using the determinants of the matrices:

$$x_{\mathbf{r}} = \begin{vmatrix} 1 & 0 & 0 \\ x_a & y_a & z_a \\ x_b & y_b & z_b \end{vmatrix} \quad y_{\mathbf{r}} = \begin{vmatrix} 0 & 1 & 0 \\ x_a & y_a & z_a \\ x_b & y_b & z_b \end{vmatrix} \quad z_{\mathbf{r}} = \begin{vmatrix} 0 & 0 & 1 \\ x_a & y_a & z_a \\ x_b & y_b & z_b \end{vmatrix}$$

CALCULATION of ANGLES:

(**mj**, **rj**) means the dihedral angle between two planes, defined by the cross products **m** x **j** and **r** x **j** of vectors which lie on the planes of interest.

Angle between Inclined Deck and Actual Deck:
(**r**, **- k**) = 23.18011
Angles at 8/12 peaks:
(**r**, **m**) = 73.83926
(**r**, **n**) = 73.83926
Angles at Beam ends:
(**r**, **i**) = 73.83924
(**r**, **j**) = 73.83924

All **R1** angles at feet = 16.161

Vectors perpendicular to Roof planes:

j x **i** = **- k** = (0, 0, -1)	**ji**, ⊥ Beam plane
m x **j** = (.55470, 0, -.83205)	**mj**, ⊥ Common plane
n x **m** = (.46154, .46154, -.69231)	**nm**, ⊥ Rafter plane
i x **n** = (0, .55470, -.83205)	**in**, ⊥ Common plane

Vectors perpendicular to Planes of Convergence:

r x **i** = (0, -3.35641, -1.01624)	**ri**, ⊥ Beam diameter
r x **j** = (3.35641, 0, 1.01624)	**rj**, ⊥ Beam diameter
r x **m** = (-.56371, 3.35641, .84556)	**rm**, ⊥ Rafter diameter
r x **n** = (-3.35641, .56371, -.84556)	**rn**, ⊥ Rafter diameter

Backing Angle complements and Backing Angles:

Dihedral angle:	**C5**	Dihedral angle:	**C5**
(**ji**, **ri**) = 73.15495	16.845	(**nm**, **rm**) = 77.82788	12.172
(**ji**, **rj**) = 73.15495	16.845	(**nm**, **rn**) = 77.82788	12.172
(**mj**, **rj**) = 73.15498	16.845	(**in**, **rn**) = 73.15498	16.845
(**mj**, **rm**) = 73.15498	16.845	(**in**, **ri**) = 73.15498	16.845

TRIGONOMETRIC SOLUTIONS:

Arcos and Arcsin Forms of Equations

cos **SS** = cos **R1** cos **C5**
The **ji** Beam plane, **mj** Common plane and **in** Common plane values may be solved using:
SS = arcos(cos 16.161 cos 16.845) = 23.180
nm Rafter plane:
SS = arcos(cos 16.161 cos 12.172) = 20.134

cos **DD** = sin **C5** / sin **SS**
ji, **mj**, and **in** planes:
DD = arcos(sin 16.845 / sin 23.180) = 42.591
nm Rafter plane:
DD = arcos(sin 12.172 / sin 20.134) = 52.226

sin **P2** = cos **R1** cos **DD**
ji, **mj**, and **in** planes:
P2 = arcsin(cos 16.161 cos 42.591) = 45.000
nm Rafter plane:
P2 = arcsin(cos 16.161 cos 52.226) = 36.039

Alternate **DD** formulas:
sin **DD** = tan **R1** / tan **SS**
tan **DD** = sin **R1** / tan **C5**

Alternate **P2** formulas:
cos **P2** = sin **R1** / sin **SS**
cos **P2** = sin **DD** / cos **C5**

SUMMARY:

Initial calculations are as per LBN #40 article. The angles discussed are with respect to the Inclined deck.

By definition, Convergence vector **r** is common to all Planes of Convergence. All Ridge vectors **i**, **j**, **m** and **n**, as well as **r**, pass through a common point at the intercept of the ridge lines.

Evaluate the 90-**R1** angles between the Ridge vectors and **r**. All **R1** angles must be equal.

Taking the cross products of successive pairs of Ridge vectors yields vectors perpendicular to the Roof planes. The cross product of each Ridge vector and **r** produces a vector perpendicular to the Planes of Convergence through the log diameters.

The dihedral angles between the Roof planes and planes of Convergence, 90-**C5**, can be calculated using the formula: $(\mathbf{a}, \mathbf{b}) = \arccos \pm \left[(\mathbf{a} \cdot \mathbf{b}) / |\mathbf{a}||\mathbf{b}|\right]$

Given values for **R1** and **C5**, the simpler trigonometric equation cos **SS** = cos **R1** cos **C5** solves the Pitch angles of the Roof planes with respect to the Inclined deck. Angles **DD** and **P2** may be solved; these angles must be equal at matching faces and edges.

For a Compound joint to be feasible:

All **R1** (Bevel angles) must be equal.

All **DD** (Miter angles) at matching faces must be equal.

The sum of the **C5** angles about a ridge line is constant.

The sum of the **P2** angles between ridges is constant.

All Ridge vector endpoints on the Inclined deck must lie on a circle.