<u>FACTORING ANGLE EQUATIONS</u>:

For convenience, algebraic names are assigned to the angles comprising the Standard Hip kernel. The names are completely arbitrary, and can vary from kernel to kernel. On the other hand, the *relationships* between the angles always remain the same. The following formulas may be used as a template or model for any kernel.

Only the distinct equations are listed; terms may be rearranged to facilitate a solution to the particular the problem at hand.

cos **R1** = cos **SS** / cos **C5**
cos **R1** = sin **P2** / cos **DD**
sin **R1** = sin **SS** cos **P2**
sin **R1** = tan **DD** tan **C5**
tan **R1** = sin **C5** / tan **P2**
tan **R1** = tan **SS** sin **DD**
cos **C5** = sin **DD** / cos **P2**
sin **C5** = sin **SS** cos **DD**
tan **C5** = tan **SS** sin **P2**
tan **P2** = cos **SS** / tan **DD**

Both formulas for tan **R1** reduce to: tan **R1** = sin **SS** sin **DD** / cos **SS**. Also, tan **P2** = cos **SS** cos **DD** / sin **DD**, and sin **C5** = sin **SS** cos **DD**. This idea of writing a formula in terms of elementary angles may be extended even further. Much as ordinary numbers may be written in terms of prime factors, each trig function of *any* angle may be expressed as a unique equation. The various formulas arise from collecting the terms in different arrangements.

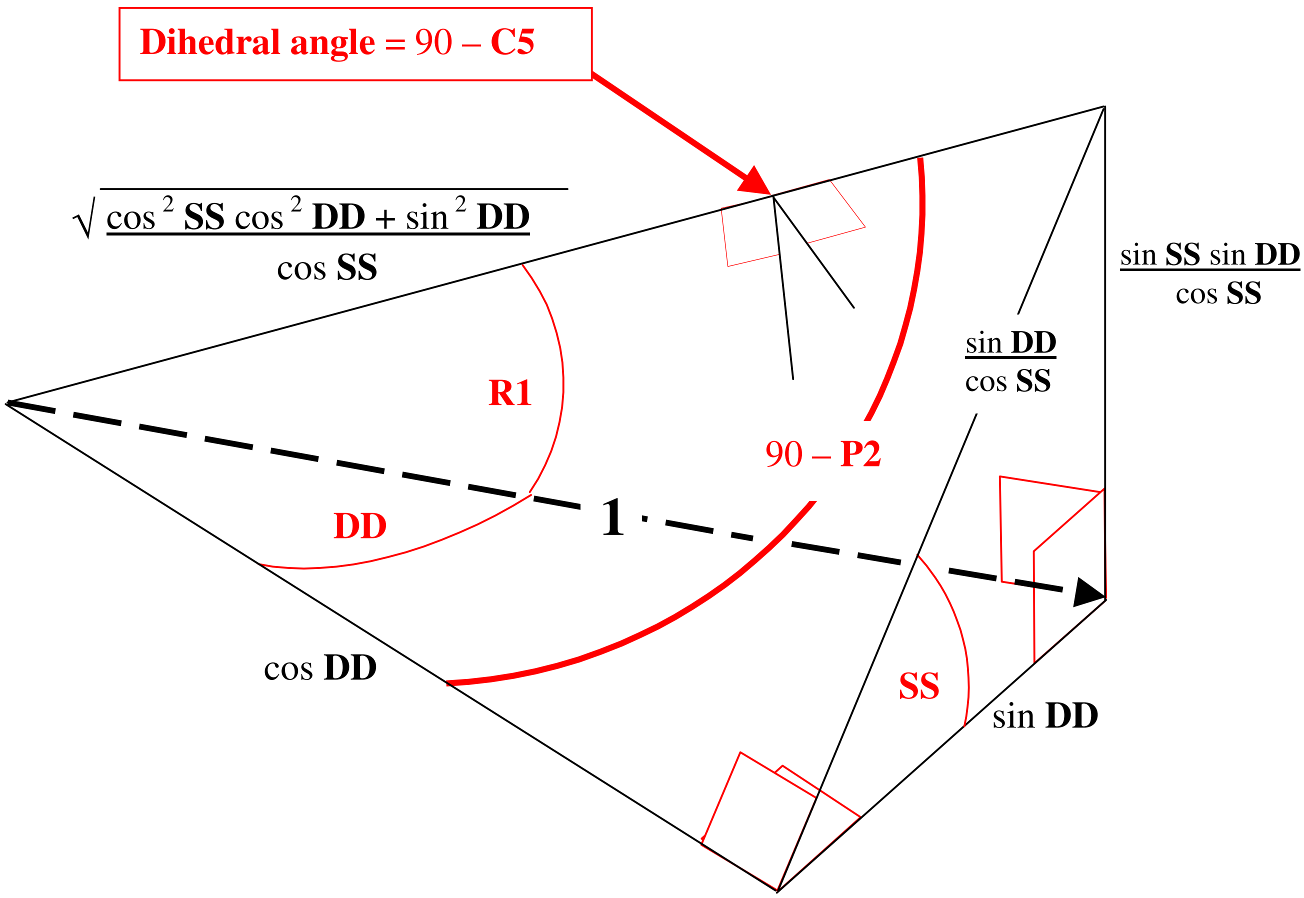
The original givens are **SS** and **DD**, and tan **θ** = sin **θ** / cos **θ**; hence, any equation may be written in terms of sin **SS**, cos **SS**, sin **DD**, and cos **DD**. Setting the Hip run of the Standard kernel equal to 1, the value representing the Hip length becomes:

$$\sqrt{(\cos^2 \mathbf{SS} \cos^2 \mathbf{DD} + \sin^2 \mathbf{DD})} \,/\, \cos \mathbf{SS}$$

Having established that it is *possible* to solve an equation using only four variables, cos **C5** may be substituted in place of the difficult radical.

STANDARD KERNEL LENGTHS:

Proportions in terms of sin **SS**, cos **SS**, sin **DD**, and cos **DD**



The initial known angles are **SS** and **DD**. The tangent of any angle may be expressed in terms of the sine and cosine. Therefore, all angle equations can be written in terms of only sin **SS**, cos **SS**, sin **DD**, and cos **DD**. To simplify collecting and cancelling terms, cos **C5** may be substituted for the radical. In addition to this natural value, angle **C5** is involved in about sixty percent of the formulas for subsequent angles, making it the logical choice for this purpose. (Linear algebraic solutions involving vector analysis use tan **R1** as a preferred value, since it lies on the *z*-axis.)

FACTORED ANGLE FORMULAS:

Solutions for the following angles were obtained utilizing kernel based trigonometry. Primary relations are **highlighted**. Note how the expressions are generally in terms of the tangent of the desired angle (compare this to the linear algebraic resolutions).

cos **P2** = sin **DD** / cos **C5**
cos **R1** = cos **SS** / cos **C5**
sin **A5B** = sin **SS** sin **DD** sin **DD** / cos **C5**
sin **A5P** = sin **SS** sin **DD** cos **DD** / cos **C5**
sin **C1** = cos **SS** cos **DD** *
sin **C2** = sin **SS** cos **SS** cos **DD** cos **DD** / cos **C5**
sin **C5** = sin **SS** cos **DD**
sin **P2** = cos **SS** cos **DD** / cos **C5**
sin **P5BV** = sin **SS** sin **DD**
sin **R1** = sin **SS** sin **DD** / cos **C5**
tan **A7** = sin **SS** cos **DD** / cos **SS**
tan **A8** = sin **SS** sin **SS** cos **SS** cos **DD** cos **DD** cos **DD** / sin **DD**
tan **A9** = sin **SS** sin **SS** sin **DD** cos **DD** / cos **SS**
tan **C5** = sin **SS** cos **DD** / cos **C5**
tan **P1** = sin **SS** cos **DD** / sin **DD**
tan **P2** = cos **SS** cos **DD** / sin **DD**
tan **P3** = sin **SS** sin **DD** cos **DD** / cos **C5** cos **C5**
tan **P5** = cos **SS** sin **DD** / cos **DD**
tan **P6** = sin **SS** cos **SS** cos **DD** cos **DD** / cos **C5** cos **C5**
tan **Q2** = sin **DD** / sin **SS** $\cos^2$ **SS** $\cos^3$ **DD**
tan **Q3** = sin **SS** / cos **SS** sin **DD**
tan **Q4** = $\sin^2$ **SS** cos **SS** sin **DD** $\cos^3$ **DD** / $\cos^3$ **C5**
tan **R1** = sin **SS** sin **DD** / cos **SS**
tan **R2** = sin **SS** cos **SS** cos **DD** cos **DD** / sin **DD**
tan **R3** = cos **C5** cos **SS** cos **DD** / sin **DD**
tan **R4B** = cos **SS** sin **DD** / cos **DD** cos **C5**
tan **R4P** = cos **SS** cos **DD** / sin **DD** cos **C5**
tan **R5B** = sin **SS** sin **DD** cos **DD** / cos **SS**
tan **R5P** = sin **SS** sin **DD** sin **DD** / cos **SS**

* Leading to: tan **P2** = sin **C1** / sin **DD**

SUBSTITUTES for RADICALS:

Formulas in terms of lowest factors

Vector analysis leads to the following relatively complex formulas. Magnitudes are selected to return trig functions of **SS** and **DD**. Values that are **highlighted** tend to repeat naturally. For convenience, at least one short trigonometric version of the expanded formula is given.

$$\cos \mathbf{A5B} = (\cos \mathbf{SS} \sqrt{1 + (\sin \mathbf{SS} \sin \mathbf{DD} \cos \mathbf{DD} / \cos \mathbf{SS})^2}\,) \div \cos \mathbf{C5}$$

$$\cos \mathbf{A5P} = (\cos \mathbf{SS} \sqrt{1 + (\sin \mathbf{SS} \sin \mathbf{DD} \sin \mathbf{DD} / \cos \mathbf{SS})^2}\,) \div \cos \mathbf{C5}$$

$$\cos \mathbf{A7} = \cos \mathbf{SS} / \cos \mathbf{P5BV} = \cos \mathbf{SS} \div \sqrt{\sin^2 \mathbf{SS} \cos^2 \mathbf{DD} + \cos^2 \mathbf{SS}}$$

$$= 1 \div \sqrt{1 + (\sin \mathbf{SS} \cos \mathbf{DD} / \cos \mathbf{SS})^2}$$

$$\cos \mathbf{A9} = \cos \mathbf{SS} \div (\cos \mathbf{C5} \sqrt{\sin^2 \mathbf{SS} \cos^2 \mathbf{DD} + \cos^2 \mathbf{SS}}\,)$$

$$\cos \mathbf{C1} = \sqrt{\sin^2 \mathbf{SS} \cos^2 \mathbf{DD} + \sin^2 \mathbf{DD}} = \sqrt{\cos^2 \mathbf{SS} \sin^2 \mathbf{DD} + \sin^2 \mathbf{SS}}$$

$$\cos \mathbf{C1} / \cos \mathbf{DD} = \sqrt{\sin^2 \mathbf{SS} + \tan^2 \mathbf{DD}} = \sqrt{\sin^2 \mathbf{SS} + \sin^2 \mathbf{DD} / \cos^2 \mathbf{DD}}$$

$$= \sqrt{\sin^2 \mathbf{SS} \cos^2 \mathbf{DD} + \sin^2 \mathbf{DD}} \div \cos \mathbf{DD}$$

$$\cos \mathbf{C1} / \sin \mathbf{SS} \sin \mathbf{DD} = \sqrt{1 + (\cot \mathbf{SS})^2 + (\cot \mathbf{DD})^2}$$

$$= \sqrt{1 + \cos^2 \mathbf{SS} / \sin^2 \mathbf{SS} + \cos^2 \mathbf{DD} / \sin^2 \mathbf{DD}}$$

$$\cos \mathbf{C2} = \sin \mathbf{C1} / \sin \mathbf{R3}$$

$$= (\sqrt{(\cos^2 \mathbf{SS} \cos^2 \mathbf{DD} + \sin^2 \mathbf{DD})^2 + (\sin \mathbf{SS} \sin \mathbf{DD} \cos \mathbf{DD})^2}\,) \div \cos \mathbf{C5}$$

$$\cos \mathbf{C5} = \sqrt{\cos^2 \mathbf{SS} \cos^2 \mathbf{DD} + \sin^2 \mathbf{DD}} = \sqrt{\sin^2 \mathbf{SS} \sin^2 \mathbf{DD} + \cos^2 \mathbf{SS}}$$

$$\cos \mathbf{P3} = \cos \mathbf{C5} / \cos \mathbf{C2} = \cos^2 \mathbf{C5} \div (\sqrt{(\cos^2 \mathbf{C5})^2 + (\sin \mathbf{SS} \sin \mathbf{DD} \cos \mathbf{DD})^2}\,)$$

FACTORED EQUATIONS (cont.):

$$\cos \mathbf{P5} = \cos \mathbf{DD} / \cos \mathbf{P5BV} = \cos \mathbf{DD} \div \sqrt{\cos^2 \mathbf{SS} \sin^2 \mathbf{DD} + \cos^2 \mathbf{DD}}$$

$$= 1 \div \sqrt{1 + (\cos \mathbf{SS} \sin \mathbf{DD} / \cos \mathbf{DD})^2}$$

$$\cos \mathbf{P5BV} = \sqrt{\sin^2 \mathbf{SS} \cos^2 \mathbf{DD} + \cos^2 \mathbf{SS}} = \sqrt{\cos^2 \mathbf{SS} \sin^2 \mathbf{DD} + \cos^2 \mathbf{DD}}$$

$$\cos \mathbf{P5BV} / \cos \mathbf{SS} \cos \mathbf{DD} = \sqrt{1 + (\tan \mathbf{SS})^2 + (\tan \mathbf{DD})^2}$$

$$= \sqrt{1 + \sin^2 \mathbf{SS} / \cos^2 \mathbf{SS} + \sin^2 \mathbf{DD} / \cos^2 \mathbf{DD}}$$

$$\cos \mathbf{P6} = \cos \mathbf{C5} / \cos \mathbf{A5P} = \frac{\cos^2 \mathbf{C5}}{\cos \mathbf{SS} \sqrt{1 + (\sin \mathbf{SS} \sin \mathbf{DD} \sin \mathbf{DD} / \cos \mathbf{SS})^2}}$$

$$\cos \mathbf{Q2} = \sin \mathbf{R2} \sin \mathbf{R3} = \frac{\sin \mathbf{SS} \cos \mathbf{DD} (\cos \mathbf{SS} \cos \mathbf{DD})^2}{\cos \mathbf{C1} \sqrt{(\cos^2 \mathbf{C5})^2 + (\sin \mathbf{SS} \sin \mathbf{DD} \cos \mathbf{DD})^2}}$$

$$\cos \mathbf{R2} = \sin \mathbf{R2} / \tan \mathbf{R2} = \sin \mathbf{DD} \div (\cos \mathbf{C5} \sqrt{\sin^2 \mathbf{SS} \cos^2 \mathbf{DD} + \sin^2 \mathbf{DD}})$$

$$\cos \mathbf{R5B} = 1 \div \sqrt{1 + (\cos \mathbf{DD} \tan \mathbf{R1})^2}$$

$$= 1 \div \sqrt{1 + (\sin \mathbf{SS} \sin \mathbf{DD} \cos \mathbf{DD} / \cos \mathbf{SS})^2}$$

$$\cos \mathbf{R5P} = 1 \div \sqrt{1 + (\sin \mathbf{DD} \tan \mathbf{R1})^2}$$

$$= 1 \div \sqrt{1 + (\sin \mathbf{SS} \sin \mathbf{DD} \sin \mathbf{DD} / \cos \mathbf{SS})^2}$$

$$\sin \mathbf{A8} = \frac{\cos \mathbf{SS} \cos \mathbf{DD} (\sin \mathbf{SS} \cos \mathbf{DD})^2}{\cos \mathbf{C5} \sqrt{(\sin^2 \mathbf{SS} \cos^2 \mathbf{DD} + \sin^2 \mathbf{DD})^2 + (\cos \mathbf{SS} \sin \mathbf{DD} \cos \mathbf{DD})^2}}$$

$$\sin \mathbf{P1} = \sin \mathbf{C5} / \cos \mathbf{C1} = \sin \mathbf{SS} \cos \mathbf{DD} \div \sqrt{\sin^2 \mathbf{SS} \cos^2 \mathbf{DD} + \sin^2 \mathbf{DD}}$$

FACTORED EQUATIONS (cont.):

$$\sin \mathbf{P4} = \tan \mathbf{P4BV} / \tan \mathbf{P1} = \frac{\cos \mathbf{SS} \sin \mathbf{DD} \cos \mathbf{DD}}{\sqrt{(\cos^2 \mathbf{C1})^2 + (\cos \mathbf{SS} \sin \mathbf{DD} \cos \mathbf{DD})^2}}$$

$$\sin \mathbf{P4BV} = \cos \mathbf{SS} \sin \mathbf{Q3} - \sin \mathbf{SS} \sin \mathbf{DD} \cos \mathbf{Q3}$$

$$\sin \mathbf{P4BV} = \frac{\sin \mathbf{SS} \cos \mathbf{SS} (\cos^2 \mathbf{DD}) (\cos^2 \mathbf{SS} + \sin^2 \mathbf{SS} \sin^2 \mathbf{DD}) \; *}{(\sin^2 \mathbf{DD} + \cos^2 \mathbf{SS} \cos^2 \mathbf{DD}) \sqrt{\sin^2 \mathbf{DD} + \sin^2 \mathbf{SS} \cos^2 \mathbf{DD}}}$$

* Collecting terms: $\sin \mathbf{P4BV} = \tan \mathbf{C1} \sin \mathbf{C5}$

$$\sin \mathbf{Q1} = \sin \mathbf{P4} / \sin \mathbf{C1} = \sin \mathbf{DD} \div \left(\sqrt{(\cos^2 \mathbf{C1})^2 + (\cos \mathbf{SS} \sin \mathbf{DD} \cos \mathbf{DD})^2}\right)$$

$$\sin \mathbf{Q3} = \sin \mathbf{SS} / \cos \mathbf{C1} = 1 \div \sqrt{1 + (\sin \mathbf{DD} / \tan \mathbf{SS})^2}$$

$$= 1 \div \sqrt{1 + (\cos \mathbf{SS} \sin \mathbf{DD} / \sin \mathbf{SS})^2}$$

$$= \sin \mathbf{SS} \div \sqrt{\cos^2 \mathbf{SS} \sin^2 \mathbf{DD} + \sin^2 \mathbf{SS}}$$

$$\sin \mathbf{R2} = \sin \mathbf{SS} \cos \mathbf{DD} \cos \mathbf{R1} \div \sqrt{\sin^2 \mathbf{SS} + \tan^2 \mathbf{DD}}$$

$$= \sin \mathbf{SS} \cos \mathbf{SS} (\cos \mathbf{DD})^2 \div (\cos \mathbf{C5} \sqrt{\sin^2 \mathbf{SS} \cos^2 \mathbf{DD} + \sin^2 \mathbf{DD}})$$

$$\sin \mathbf{R3} = \frac{\cos \mathbf{C5} \cos \mathbf{SS} \cos \mathbf{DD}}{\sqrt{(\cos^2 \mathbf{SS} \cos^2 \mathbf{DD} + \sin^2 \mathbf{DD})^2 + (\sin \mathbf{SS} \sin \mathbf{DD} \cos \mathbf{DD})^2}}$$

$$\sin \mathbf{R4B} = \sin \mathbf{DD} \div \sqrt{1 + (\sin \mathbf{SS} \sin \mathbf{DD} \cos \mathbf{DD} / \cos \mathbf{SS})^2}$$

$$\sin \mathbf{R4P} = \cos \mathbf{DD} \div \sqrt{1 + (\sin \mathbf{SS} \sin \mathbf{DD} \sin \mathbf{DD} / \cos \mathbf{SS})^2}$$

$$\tan \mathbf{P4} = \tan \mathbf{C1} \cos \mathbf{P1} = \cos \mathbf{SS} \cos \mathbf{DD} \sin \mathbf{DD} / (\sin^2 \mathbf{SS} \cos^2 \mathbf{DD} + \sin^2 \mathbf{DD})$$

FACTORED EQUATIONS (cont.):

$$\tan \mathbf{P4BV} = \sin \mathbf{A8} / \tan \mathbf{C5} = \frac{\sin \mathbf{SS} \cos \mathbf{SS} \cos^2 \mathbf{DD}}{\sqrt{(\cos^2 \mathbf{C1})^2 + (\cos \mathbf{SS} \sin \mathbf{DD} \cos \mathbf{DD})^2}}$$

$$\tan \mathbf{Q1} = \tan \mathbf{P4} / \sin \mathbf{P4BV} = \sin \mathbf{DD} \div (\sin \mathbf{SS} \cos \mathbf{DD} \sqrt{\sin^2 \mathbf{SS} \cos^2 \mathbf{DD} + \sin^2 \mathbf{DD}}\,)$$

The more complex the required joint, the more involved the calculations for the angles become. Several substitutions for **C1** and **C5** have already been made for the sake of clarity, and note the formula for **Q4** below:

$$\cos \mathbf{Q4} = \cos \mathbf{P6} / \cos \mathbf{C2}$$

$$= \cos \mathbf{P3} / \cos \mathbf{A5P}$$

$$= \frac{\cos^3 \mathbf{C5}}{\cos \mathbf{SS} \sqrt{1 + (\sin \mathbf{SS} \sin \mathbf{DD} \cos \mathbf{DD} / \cos \mathbf{SS})^2} \sqrt{(\cos^2 \mathbf{C5})^2 + (\sin \mathbf{SS} \sin \mathbf{DD} \cos \mathbf{DD})^2}}$$

The angles that are listed thus far have been solved in terms of at least two of their trig functions, therefore a resolution in terms of the remaining function is possible. **R6P**, **R6PBV**, **R7**, **R7BV**, **VC**, and **VP** remain unsolved. Vector analysis becomes very involved; there is sufficient information given above to simply substitute in the trigonometric formulas of these angles to obtain expressions in terms of only four variables.